Chapter 1: Real Numbers Questions and Answers

I. Multiple Choice Questions (MCQ)

Q. No.	Question	Correct Answer	Explanation
1	Euclid's division algorithm can be applied to: (a) only positive integers (b) only negative integers (c) all integers (d) all integers except 0.	**(a) only positive integers**	Euclid's algorithm is defined for positive integers.
2	For some integer m, every even integer is of the form: (a) m (b) $m+1$ (c) **2m** (d) $2m+1$	**(c) 2m**	An even integer is a multiple of 2.
3	For some integer q, every odd integer is of the form: (a) 2q (b) **2q+1** (c) q (d) $q+1$	**(b) 2q+1**	An odd integer is one more than an even integer.
4	If the HCF of 65 and 117 is expressible in the form $65m - 117$, then the value of m is: (a) 1 (b) **2** (c) 3 (d) 4	**(b) 2**	HCF(65, 117) = 13. $65m - 117 = 13 \implies m = \mathbf{2}$.
5	If the HCF of 85 and 153 is expressible in the form $85m - 153$, then the value of m is: (a) 1 (b) 4 (c) 3 (d) **2**	**(d) 2**	HCF(85, 153) = 17. $85m - 153 = 17 \implies m = \mathbf{2}$.
6	If two positive integers p and q can be expressed as $p = ab^2$ and $q = a^3b$; a, b being prime numbers, then LCM (p, q) is: (a) ab (b) a^2b^2 (c) a^3b^2 (d) a^3b^3	**(c) a^3b^2**	Highest powers: a^3 and b^2.
7	If two positive integers p and q can be expressed as $p = ab^2$ and $q = a^2b$; a, b being prime numbers, then LCM (p, q) is: (a) a^2b^2 (b) ab (c) a^3b^3 (d) a^3b^2	**(a) a^2b^2**	Highest powers: a^2 and b^2.
8	If two integers a and b are written as $a = x^3y^2$ and $b = xy^4$; x, y are prime numbers, then H.C.F. (a, b) is: (a) x^3y^3 (b) x^2y^2 (c) xy (d) xy^2	**(d) xy^2**	Lowest powers: x^1 and y^2.
9	The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is: (a) 10 (b) 100 (c) 504 (d) **2520**	**(d) 2520**	LCM(1 to 10) = $\mathbf{2520}$.
10	$7 \times 11 \times 13 \times 15 + 15$ is: (a) **composite number** (b) prime number (c) neither composite nor prime (d) none of these	**(a) composite number**	Can be factored as $15 \times (\dots)$, so it is composite.
11	$1.23\overline{48}$ is: (a) an integer (b) an irrational number (c) **a rational number** (d) none of these	**(c) a rational number**	Non-terminating, repeating decimal.
12	$2.\overline{35}$ is: (a) a terminating decimal (b) **a rational number** (c) an irrational number (d) both (a) and (c)	**(b) a rational number**	Non-terminating, repeating decimal.
13	$3.24636363...$ is: (a) a terminating decimal number (b) a non-terminating repeating decimal number (c) a rational number (d) **both (b) and (c)**	**(d) both (b) and (c)**	Non-terminating, repeating decimal, thus rational.
14	Euclid's division lemma states that for two positive integers a and b, there exist unique integers q and r such that $a = bq + r$; where: (a) $0 < r \le b$ (b) $1 < r < b$ (c) $0 < r < b$ (d) $0 \le r < b$	**(d) $0 \le r < b$**	Remainder r must be non-negative and less than b.

16	HCF and LCM of a and b are 19 and 152 respectively. If $a = 38$ then b is equal to: (a) 152 (b) 19 (c) 38 (d) **76**	**(d) 76**	$38 \times b = 19 \times 152 \implies b = \mathbf{76}$.
17	The largest number which divides 71 and 126, leaving remainders 6 and 9 respectively is: (a) 1750 (b) **13** (c) 65 (d) 875	**(b) 13**	HCF(71-6, 126-9) = HCF(65, 117) = $\mathbf{13}$.
20	Which of the following rational numbers have a terminating decimal expansion? (a) $\frac{125}{441}$ (b) $\frac{77}{210}$ (c) $\frac{15}{1600}$ (d) $\frac{129}{2^2 \times 5^2 \times 7^2}$	**(c)** $\frac{15}{1600}$	Denominator $1600 = 2^6 \times 5^2$ has only 2 and 5 as factors.
24	The rational number in the following is... ($\sqrt{3}, \sqrt{\mathbf{4}}, \sqrt{5}, \sqrt{7}$)	$\sqrt{4}$ **(or 2)**	$\sqrt{\mathbf{4}} = \mathbf{2}$, which is rational.
25	LCM of 2 and 3 is...	**6**	LCM of two primes is their product.

II. Short/Long Answer Questions

A. Questions using Euclid's Division Lemma/Algorithm

5. Question: Use Euclid's division lemma to show that square of any positive integer is either of form $3m$ or $3m+1$ for some integer m.

Answer: Let a be any positive integer. By Euclid's Division Lemma with divisor $b=3$, a is $3q$, $3q+1$, or $3q+2$.

- If $a=3q \implies a^2=9q^2=3(\mathbf{3q^2})=\mathbf{3m}$.
- If $a=3q+1 \implies a^2=9q^2+6q+1=3(\mathbf{3q^2+2q})+1=\mathbf{3m+1}$.
- If $a=3q+2 \implies a^2=9q^2+12q+4=3(\mathbf{3q^2+4q+1})+1=\mathbf{3m+1}$.

6. Question: Use Euclid's division algorithm to find the HCF of 10224 and 9648.

Answer:

1. $10224=9648\times 1+576$
2. $9648=576\times 16+432$
3. $576=432\times 1+144$
4. $432=144\times 3+0$

$$\mathbf{HCF(10224,9648)}=\mathbf{144}$$

B. Questions using Fundamental Theorem of Arithmetic (HCF/LCM)

4. Question: Check whether 6^n can end with the digit 0 for any natural number n.

Answer: For a number to end with **0**, its prime factors must include **2 and 5**. The prime factorization of 6^n is:

$$6^n=(2\times 3)^n=2^n\times 3^n$$

Since $\mathbf{5}$ is not a prime factor of 6^n, 6^n **cannot end with the digit 0**.

6. Question: Sonia takes 18 minutes to drive one round... while Ravi takes 12 minutes... After how many minutes will they meet again at the starting point?

Answer: They will meet at the **Least Common Multiple (LCM)** of their timings.

- $18=2\times 3^2$
- $12=2^2\times 3^1$
- $\mathrm{LCM}(18,12)=2^2\times 3^2=4\times 9=\mathbf{36}$.

They will meet again after **36 minutes**.

C. Proof of Irrationality (Using Unicode √)

2. Question: Prove that $\sqrt{\mathbf{5}}$ is irrational.

Answer (Proof by Contradiction):

1. Assume $\sqrt{\mathbf{5}} = \frac{a}{b}$ (rational, a, b co-prime).
2. $5 = \frac{a^2}{b^2} \implies a^2 = 5b^2. \implies a$ is divisible by 5.
3. Let $a = 5c$. Substituting: $(5c)^2 = 5b^2 \implies 25c^2 = 5b^2 \implies 5c^2 = b^2$.
4. b^2 is divisible by 5 $\implies b$ is divisible by 5.
5. Since both a and b are divisible by 5, this **contradicts** the assumption that they are co-prime. Therefore, **$\sqrt{\mathbf{5}}$ is an irrational number**.

3. Question: Prove that $3\sqrt{\mathbf{5}} - 2$ is an irrational number.

Answer (Proof by Contradiction):

1. Assume $3\sqrt{\mathbf{5}} - 2$ is rational, so $3\sqrt{\mathbf{5}} - 2 = k$.
2. Isolating $\sqrt{\mathbf{5}}$: $\sqrt{\mathbf{5}} = \frac{k+2}{3}$.
3. Since $k, 2,$ and 3 are rational, the entire expression $\frac{k+2}{3}$ is **rational**.
4. This implies $\sqrt{\mathbf{5}}$ is rational, which is a **contradiction**. Therefore, **$3\sqrt{\mathbf{5}} - 2$ is an irrational number.**